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# Model for simulating uncertainties of TOC from multispectral measurements

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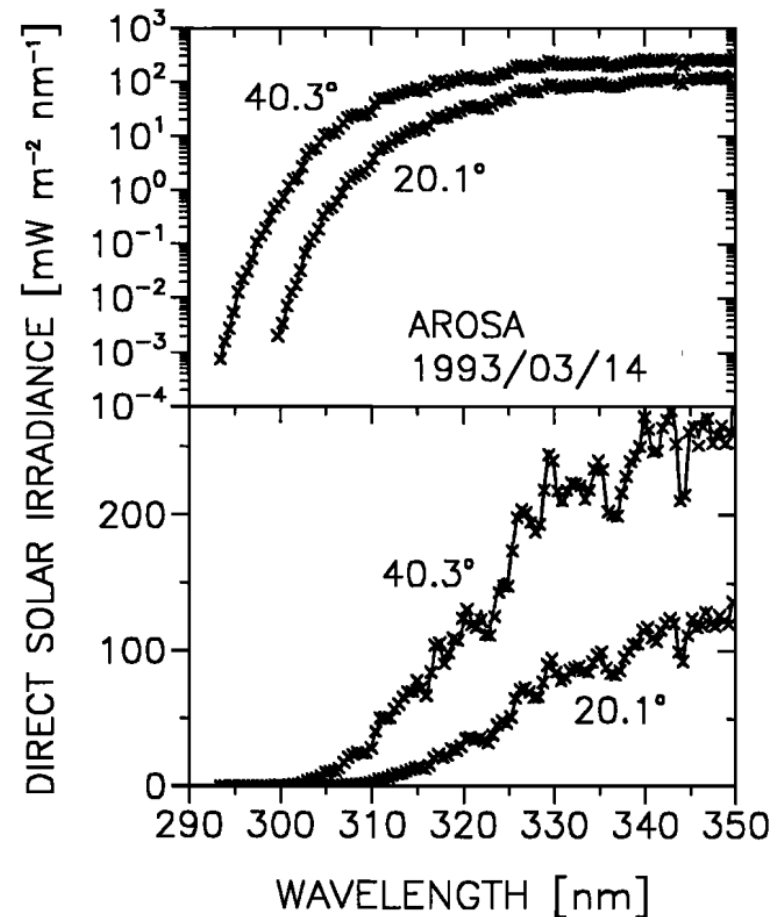
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# Deriving ozone from measured spectrum

- High resolution spectral measurements of direct solar UV irradiance
- $\lambda = [290, 350]$  nm,  $\Delta\lambda = 0.5$  nm
- Total atmospheric ozone amount determined by fitting model calculations to the measured spectra
- M. Huber, M. Blumthaler, and W. Ambach, "Total atmospheric ozone determined from spectral measurements of direct solar UV irradiance," *Geophys. Res. Lett.* **22**, 53-56 (1995).

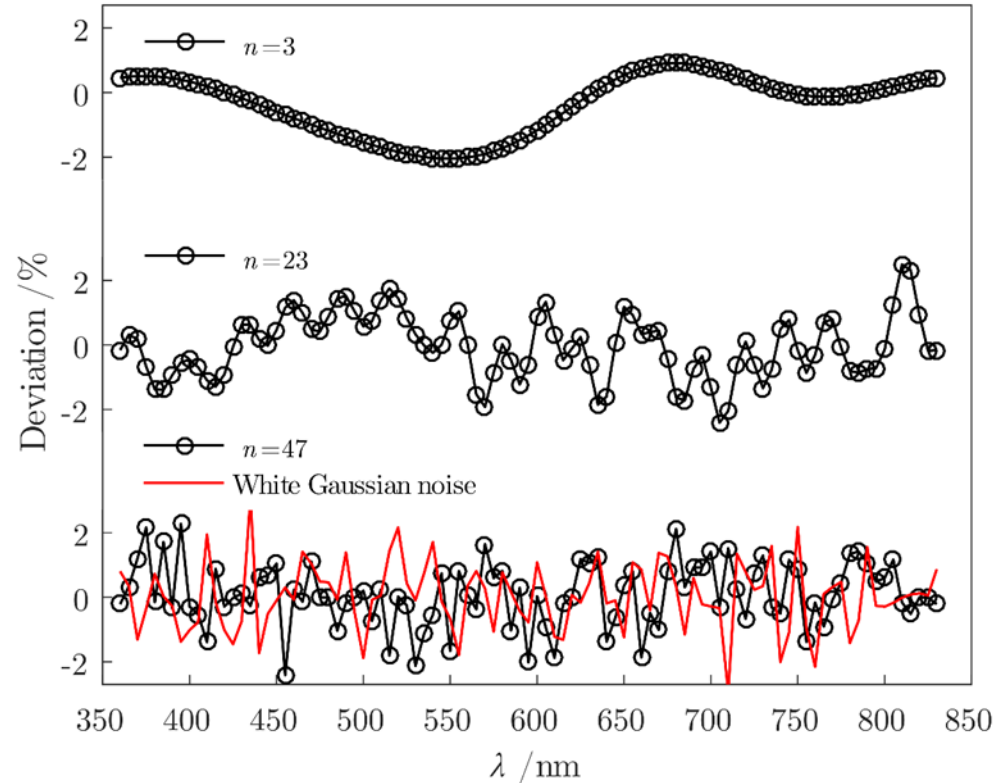


# Model calculation

- Extraterrestrial solar spectrum  $E_{Ext}(\lambda)$  used as starting point
- Effect of atmosphere is modeled as
- $E(\lambda) = E_{Ext}(\lambda) \cdot e^{-\tau(\lambda) \cdot m}$ , where
- $\tau(\lambda) = \alpha_{O_3}(\lambda) \cdot TOC + d_{Rayleigh}(\lambda) + d_{aod}(\lambda)$ 
  - $m$  is the relative air mass
  - $\tau(\lambda)$  is the optical depth of the atmosphere
  - $\alpha_{O_3}(\lambda)$  is the ozone absorption cross section
  - $TOC$  is the total ozone column
  - $d_{Rayleigh}(\lambda)$  is the Rayleigh scattering optical depth
  - $d_{aod}(\lambda) = \beta \cdot \lambda^{-1.4}$  is the aerosol optical depth,  $\lambda$  in  $\mu\text{m}$ .
- In the analysis,  $TOC$  and  $\beta$  are varied to minimize difference between measured and modeled irradiances using least squares fitting
- Convolution accounted for.
- Simplified model to speed up calculations.

# Correlations in measurement data

- Uncertainty of spectral irradiance typically includes:
  - Noise
  - Uncertainty of standard lamp
  - Transfer uncertainties of calibrations
  - Residuals of corrections
  - Interpolation errors
- Uncertainties may hide spectrally varying errors due to correlations
- Typically it is assumed that spectral irradiance data are uncorrelated, noise
- Correlated data behave differently in integrations or models
- Correlation matrix needed (if available). If not, correlations need to be estimated.



# Method for analyzing possible correlations

- Orthogonal base functions formed as a series of Sines with  $\sigma = 1$ , limits  $\lambda_1$  and  $\lambda_2$  depend on application.

$$\begin{cases} f_i(\lambda) = \sqrt{2} \sin \left[ i \left( 2\pi \frac{\lambda - \lambda_1}{\lambda_2 - \lambda_1} \right) + \phi_i \right] \\ f_0(\lambda) = 1 \end{cases}$$

- An error function is formed by combining  $N+1$  first terms with varying weights

$$\delta(\lambda) = \sum_{i=0}^N \gamma_i f_i(\lambda)$$

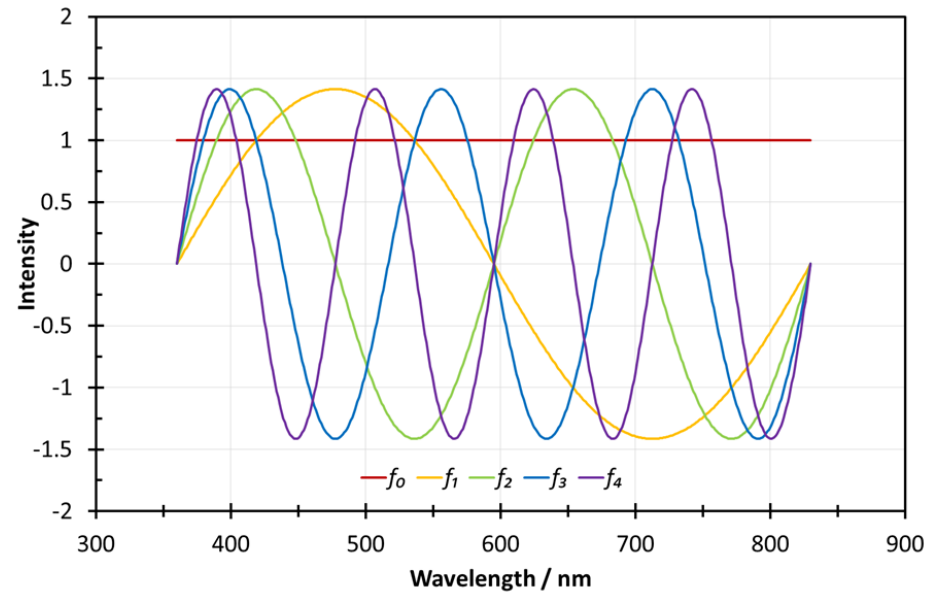
- Weights  $\gamma_i$  are chosen randomly from the surface of  $N+1$  dimensional sphere to keep variance 1.

- Data are disturbed as

$$E_e(\lambda) = [1 + \delta(\lambda) u_c(\lambda)] E(\lambda)$$

and used to calculate desired results CCT or TOC.

- Results are repeated to calculate standard deviations and  $N$  is varied.



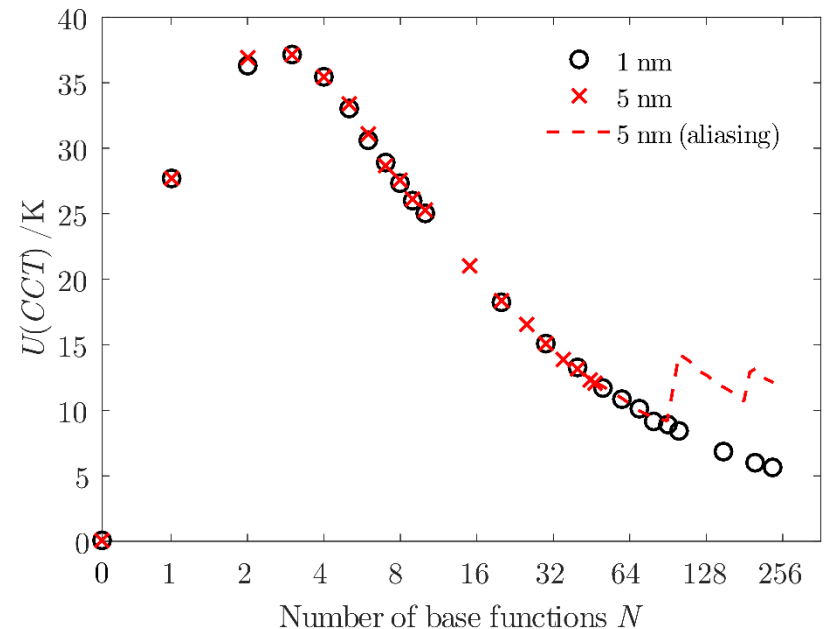
$$P = \{\gamma_0, \gamma_1, \dots, \gamma_N\}$$

=

$$\left\{ \frac{\gamma_0}{\sqrt{\gamma_0^2 + \gamma_1^2 + \dots + \gamma_N^2}}, \frac{\gamma_1}{\sqrt{\gamma_0^2 + \gamma_1^2 + \dots + \gamma_N^2}}, \dots, \frac{\gamma_N}{\sqrt{\gamma_0^2 + \gamma_1^2 + \dots + \gamma_N^2}} \right\}$$

# Correlated color temperature $CCT = 2856$ K of a lamp

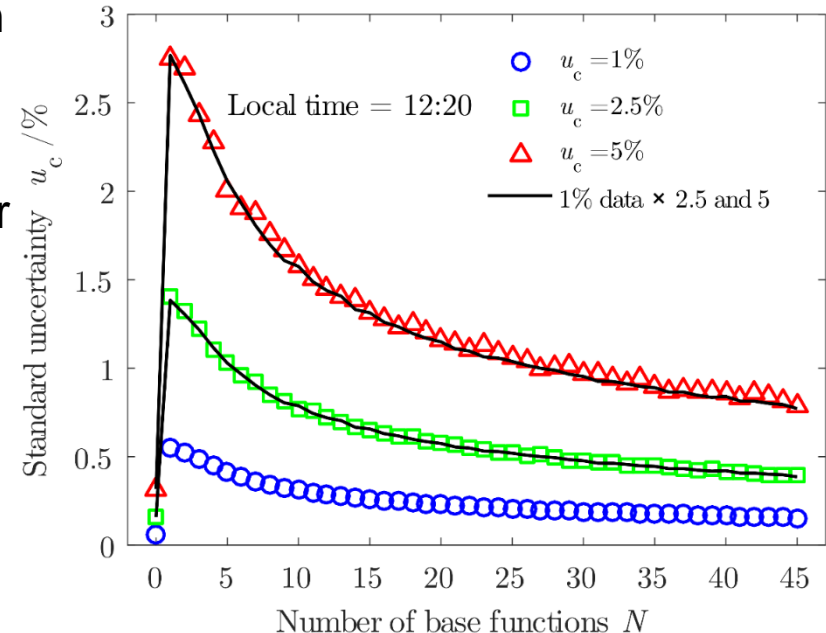
- $CCT$  is the temperature of a Planckian radiator whose color coordinates  $(u, v)$  are closest to the color coordinates of the lamp.
- Assuming 1% standard uncertainty at each wavelength.
- Obtained graph gives three important values
  - Uncertainty with full correlation  $N = 0$ ,  $U_{FC} = 0$ , in the case of  $CCT$
  - Uncertainty with no correlation at Nyquist criterion  $N = 235$ ,  $U_{NC} = 5.6$  K
  - Uncertainty with unfavorable correlation  $N = 3$ ,  $U_{UC} = 37.2$  K. Worst case with 6 sign change within wavelength range.
- Unambiguous value can not be derived if correlations are unknown. One possibility to use average of the three values  $U = 14$  K.



P. Kärhä, A. Vaskuri, H. Mäntynen, N. Mikkonen, and E. Ikonen, "Method for estimating effects of unknown correlations in spectral irradiance data on uncertainties of spectrally integrated colorimetric quantities" (submitted).

# Uncertainty of $TOC$ at noon

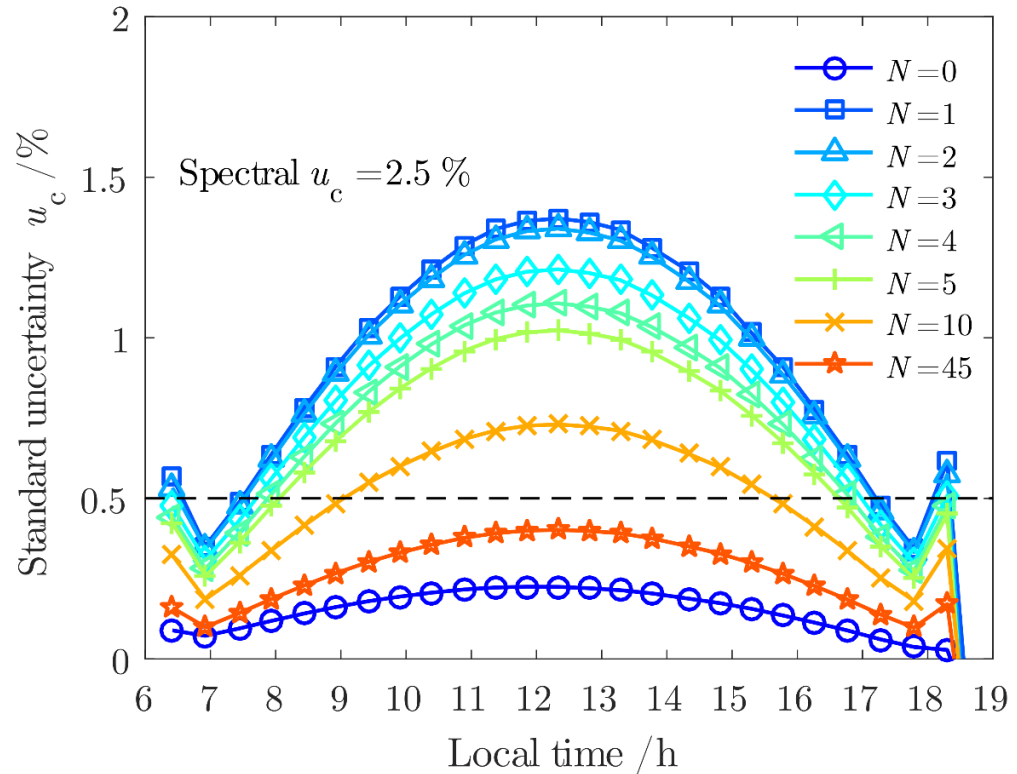
- Spectra measured in Mauna Loa, USA on Nov 30, 2001, 6:14 – 18:54 analyzed.
- $TOC$  was  $\sim 264$  DU.
- Uncertainties at noon (12:20) analyzed for three uncertainties  $u_c$  ( $k = 1$ ) = 1%, 2.5%, 5%
  - Maximum uncertainty at  $N = 1$ . Obviously a slope produces highest uncertainty.
  - Process is linear and scalable! Data can be used as sensitivities when analyzing uncertainties.
- For  $U$  ( $k = 2$ ) = 5%,  $U_{FC} = 0.3\%$ ,  $U_{NC} = 0.8\%$ ,  $U_{UC} = 2.75\%$
- Assuming again average of the three yields  $U = 1.3\%$  (3.4 DU)





# Uncertainty of TOC during the day

- Uncertainties of  $TOC$  analyzed throughout the day assuming  $u_c (k = 1) = 2.5\%$ .
- Sensitivity of  $O_3$  uncertainty on uncertainty in irradiance is highest at Noon and lowest in the evening and morning due to length of the air mass.
- On the other hand, uncertainties of spectral irradiance are also higher in the evening and morning due to lower signal levels.



# Conclusions

- In order to calculate uncertainties of quantities derived from spectra (*CCT*, *TOC*), uncertainty of spectral irradiance is not enough. Correlation data is needed as well.
- Correlation may decrease but also increase the uncertainties of derived quantities.
- We have presented a Monte Carlo based method for studying effects that possible correlations have on uncertainties of quantities derived from spectra. Can be used if correlation data is not available.
- Gives three useable values: Uncertainty assuming full correlation, Uncertainty assuming no correlations, and Uncertainty assuming an unfavorable case of correlations. The three values give limits within which the uncertainty must reside.
- *TOC* is most sensitive to a slope-type of error in spectral irradiance, whereas *CCT* suffers mostly from an error with 6 sign changes within the wavelength range of interest.

EMRP ENV59 ATMOZ “Traceability for atmospheric total column ozone”

