Simulating effects of unknown correlations in spectral data on uncertainties of TOC derived from multispectral measurements

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Uncertainty of spectrally derived quantities

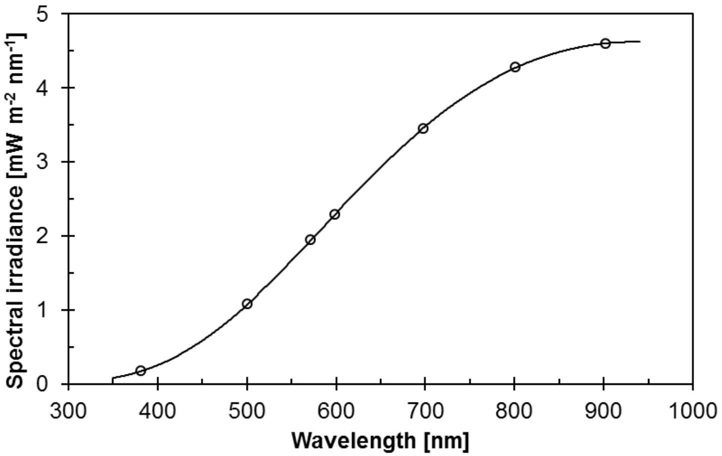
- Quantities such as Color Coordinates *x* and *y*, color temperature *CCT*, and amount of ozone *TOC* are calculated using measured spectra
- Uncertainties of spectral irradiance values at different wavelengths are correlated, but mostly we do not know how
- New approach: form spectral functions that simulate different scenarios that the correlations may have
- Monte Carlo analysis with the functions gives uncertainty values
- Two steps:
  - First develop methods for *x*, *y* and *CCT* (more familiar, possiblity to compare)
  - Extend to TOC
- Progress with colorimetry

Preliminary work with FEL lamps

- Analyse uncertainties of color coordinates of Aalto spectral irradiance lamps
- *x, y* derived from spectrum as integrals
- Analytical function for CCT [C. S. McCamy, “Correlated color temperature as an explicit function of chromaticity coordinates,” *Color Res. Appl.* **17**, 142 – 144 (1992)].
- Form a distorted spectrum

$$E_e(\lambda) = \big(1 + \varepsilon\delta(\lambda)u_E(\lambda)\big)E(\lambda)$$

  - $u_E(\lambda)$ is the relative uncertainty of $E(\lambda)$
  - $\delta(\lambda)$ is an error function with $\sigma = 1$.
  - $\varepsilon \sim N(0,1)$ is a Monte Carlo variable
- The error function is formed as sum of cosines



$$X = \int_{\lambda_1}^{\lambda_2} E(\lambda)\bar{x}d\lambda,$$

$$Y = \int_{\lambda_1}^{\lambda_2} E(\lambda)\bar{y}d\lambda,$$

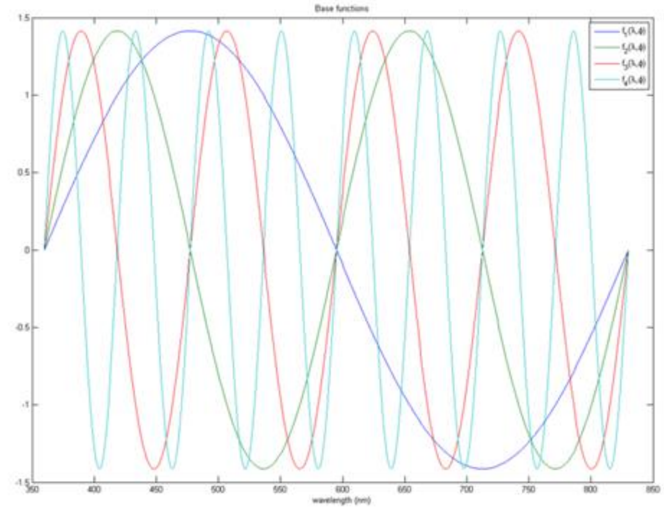
$$Z = \int_{\lambda_1}^{\lambda_2} E(\lambda)\bar{z}d\lambda,$$

$x = X/(X + Y + Z)$, and

$y = X/(X + Y + Z)$,

Error function from orthogonal base-functions in $[\boldsymbol{\lambda_1}, \boldsymbol{\lambda_2}]$

- The error function is formed as $\delta(\lambda) = \gamma_1 f_1(\lambda) + \gamma_2 f_2(\lambda) + \gamma_3 f_3(\lambda) + \gamma_4 f_4(\lambda)$
- γ_1 - γ_N are Monte Carlo variables.
- To get standard uncertainties, it must be that
$$\gamma_1^2 + \gamma_2^2 + \gamma_3^2 + \gamma_4^2 = 1$$
- This can be accomplihed by taking a random point in *N*-dimensional spherical coordinate system, and using its *x*, *y*, *z*, … coordinates as the weights
- The base functions have been chosen so that their *RMS*-value is 1 in $[\boldsymbol{\lambda_1}, \boldsymbol{\lambda_2}]$
- $E_e(\lambda) = \big(1 + \varepsilon\delta(\lambda)u_E(\lambda)\big)E(\lambda)$



$$f_1(\lambda) = \frac{2}{\sqrt{\pi}} \sin\left(1\left(2\pi\frac{\lambda - \lambda_1}{\lambda_2 - \lambda_1} + \phi\right)\right)$$

$$f_2(\lambda) = \frac{2}{\sqrt{\pi}} \sin\left(2\left(2\pi\frac{\lambda - \lambda_1}{\lambda_2 - \lambda_1} + \phi\right)\right)$$

$$f_3(\lambda) = \frac{2}{\sqrt{\pi}} \sin\left(4\left(2\pi\frac{\lambda - \lambda_1}{\lambda_2 - \lambda_1} + \phi\right)\right)$$

$$f_4(\lambda) = \frac{2}{\sqrt{\pi}} \sin\left(8\left(2\pi\frac{\lambda - \lambda_1}{\lambda_2 - \lambda_1} + \phi\right)\right)$$

Preliminary results

- Simulation of uncertainties of colorimetric properties of Aalto lamps
- With 10 000 runs
- Individual base functions give maxima at $f_1 - f_2$ and reduce to negligible at $\sim f_5$
- Cumulative uncertainties sound reasonable

		Uncertainty					
	Value	f1	f2	f3	f4	f5	f6
x	0.43060	0,00058	0,00055	0,00025	0,00002	0,00000	0,00000
y	0.40231	0,00027	0,00056	0,00019	0,00002	0,00000	0,00000
CCT / K	3101.7 K	9,56	11,47	5,49	0,41	0,05	0,01

		Uncertainty					
	Value	f1	f1 + f2	f1 + f2 + f3	f1 + f2 + f3 + f4	f1 + f2 + f3 + f4 + f5	f1 + f2 + f3 + f4 + f5 + f6
x	0.43060	0,00058	0,00056	0,00048	0,00042	0,00037	0,00034
y	0.40231	0,00027	0,00045	0,00038	0,00033	0,00030	0,00027
CCT / K	3101.7 K	9,56	10,56	9,30	8,07	7,08	6,58

Plan for further work

- Half wave, quarter wave, and full correlation should be considered as well and added to the model
- Simulations give maksimum uncertainties and conditions leading to them
  - How will the maxima convert to standard uncertainties?
  - How should the components to include be selected?
    - Go through all scenarios?
    - Select the most likely base functions?
- Compare with earlier work of Jim Gardner of CSIRO
- After knowing that the model works, apply to Ozone measurements
  - How should the base functions then be selected? The wavelength range 290 – 320 is significantly narrow, there can not be many sign changes.